Pearson Edexcel

Mark Scheme (Results)

January 2024

Pearson Edexcel International Advanced Level In Pure Mathematics P3 (WMA13) Paper 01

Question Number	Scheme	Marks
1(a)	$(-2, -3)$	B1
		(1)
(b)	$(-3, -9)$	B1B1
		(2)
(c)	$(-4, 3)$	B1
		(1)
		(4 marks)

In all parts, if more than 1 coordinate pair is offered then mark the "final" pair.

Accept missing brackets as long as the intention is clear e.g. $-2, -3$ **or** $(-2, -3$

(a)

B1: $(-2, -3)$. Allow $x = -2$, $y = -3$.

(b)

B1: One correct coordinate.

May be seen as part of a coordinate pair or written separately as $x = -3$ or $y = -9$.

B1: $(-3, -9)$. Allow $x = -3$, $y = -9$.

(c)

B1: $(-4, 3)$. Allow $x = -4$, $y = 3$.

Question Number	Scheme	Marks
2(a)	$\left(\text{f}(2)=\right)2^4-5(2)^2+4(2)-7=-3<0$ **and** $\left(\text{f}(3)=\right)3^4-5(3)^2+4(3)-7=41>0$	M1
	There is a change of sign and $\text{f}(x)$ is continuous so there is a root (in the interval) *	A1*
		(2)
(b)	$x^3=\dfrac{5x^2-4x+7}{x}\Rightarrow x=\sqrt[3]{\dfrac{5x^2-4x+7}{x}}$ *	B1*
		(1)
(c)(i)	$x_2=\sqrt[3]{\dfrac{5(2)^2-4(2)+7}{2}}$	M1
	$=\text{awrt }2.1179$	A1
(ii)	$\alpha=\text{awrt }2.1565$	A1
		(3)
		(6 marks)

(a)

M1: Attempts both $\text{f}(2)$ and $\text{f}(3)$ or a narrower interval that contains the root 2.1565…and considers the signs. Note showing $\text{f}(2)\times\text{f}(3)<0$ or "< 0", "> 0" beside each appropriate root is a consideration of signs and is also sufficient for the "sign change" part of reasoning for the A1.
For the substitution, need to see the values substituted or at least one of f(2) or f(3) correct.

A1*: This mark requires:

- both $\text{f}(2)$ and $\text{f}(3)$ correct e.g. f(2) = –3 and f(3) = 41
- a reference to the sign change
- a reference to continuity
- a (minimal) conclusion

For **sign change**, allow equivalent statements e.g. f(2).f(3) < 0, f(2) < 0 < f(3) etc.
For **continuity**, allow just "continuous" and allow "assuming it is continuous" and "continuous equation" but do **not** allow incorrect statements such as "*x* is continuous", "the interval is continuous", "there is a change of sign therefore it is continuous"
For the **conclusion**, allow e.g. ✓, #, QED, hence shown, hence root, so it crosses the *x*-axis, etc., but **not** incorrect statements e.g. so there is a root in the interval $[-3, 41]$

You may have to use your judgement to decide if the A1 conditions are satisfied.

Example: f(2) = –3, f(3) = 41, there is a change of sign hence f(*x*) has a continuous root in [2, 3]
Scores M1A0 – i.e. it is not the root that is continuous

(b)

B1*: Proceeds to given answer with no errors and with $x^3 = ...$ seen at some stage.

Condone omission of the fraction bar e.g. $x = \sqrt[3]{\begin{matrix}5x^2 - 4x + 7 \\ x\end{matrix}}$ as long as there are no algebraic errors. Be tolerant if the radical does not fully encompass the expression but score B0 if the expression is clearly incorrect e.g. $x = \frac{\sqrt[3]{5x^2 - 4x + 7}}{x}$ i.e. where the cube root clearly sits on top of the fraction bar.

Condone working backwards as long as there is a (minimal) conclusion e.g.

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}} \Rightarrow x^3 = \frac{5x^2 - 4x + 7}{x} \Rightarrow x^4 - 5x^2 + 4x - 7 = 0$$ ✓, #, QED, hence shown, etc.

(c)(i)

M1: Attempts to find x_2 using the given iteration formula. Allow for sight of $\sqrt[3]{\frac{5(2)^2 - 4(2) + 7}{2}}$

Must see the correct index and not $\sqrt{\frac{5(2)^2 - 4(2) + 7}{2}}$ unless the "3" is implied by their value(s). May be implied by awrt 2.118 or awrt 2.147 $(= x_3)$

A1: awrt 2.1179 (apply isw if necessary)

(ii)

A1: awrt 2.1565 (provided M1 scored in (c)(i)) (apply isw if necessary)

Question Number	Scheme	Marks
3(a)	$\log_{10} D = 1.04 + 0.38t \Rightarrow D = 10^{1.04+0.38t}$ **or** $a = 10^{1.04}$ or $b = 10^{0.38}$	M1
	$a =$ awrt 10.96 **or** $b =$ awrt 2.399	A1
	$D = 10.96 \times 2.399^t$	A1
		(3)
(b)	$45000 = "10.96" \times "2.399"^T \Rightarrow T = ...$ or $\log_{10} 45000 = 1.04 + 0.38T \Rightarrow T = ...$	M1
	awrt 9.51	A1
		(2)
(c)	$D = "10.96" \times "2.399"^{12} \Rightarrow D = ...$ or $\log_{10} D = 1.04 + 0.38 \times 12 \Rightarrow D = ...$ or $350000 = "10.96" \times "2.399"^t \Rightarrow t = ...$ or $\log_{10} 350000 = 1.04 + 0.38 \times t \Rightarrow t = ...$	M1
	D = awrt (£)400 000 $\Rightarrow$ yes or t = awrt 11.9 $\Rightarrow$ yes	A1
		(2)
		(7 marks)

(a)

M1: A correct application of log laws to obtain $D = 10^{1.04+0.38t}$ or forms one correct equation for a or b e.g. $a = 10^{1.04}$ or $b = 10^{0.38}$. May be implied by a = awrt 11 or b = awrt 2.4

A1: One of $a =$ awrt 10.96, $b =$ awrt 2.399. May be seen embedded in their equation.

A1: Correct **equation** with correct awrt values e.g. $D = (awrt\,10.96) \times (awrt\,2.399)^t$

Do <u>not</u> allow recovery from incorrect work e.g.

$$\log_{10} D = 1.04 + 0.38t \Rightarrow D = 10^{1.04} + 10^{0.38t} \Rightarrow a = 10.96,\ b = 2.399 \Rightarrow D = 10.96 \times 2.399^t$$

Scores M1A1A0

Note that *a* and *b* may be found in other ways e.g.

$$t = 0 \Rightarrow a = D = 10^{1.04},\ t = 1 \Rightarrow b = \frac{D}{a} = \frac{10^{1.42}}{10^{1.04}} = 10^{0.38}$$

(b)

M1: This mark is awarded for proceeding to T = … or t = … where … may be a numerical expression e.g. ln(…) from one of:

- substituting $D = 45000$ into their $D = "10.96" \times "2.399"^T$ (Must be of this form)
- substituting $D = 45000$ into $\log_{10} D = 1.04 + 0.38T$ (Must be the given equation)

A1: awrt 9.51 (months) (Ignore labelling and just look for the value)
Units are **not** required but if any are given they must be correct.

Allow recovery from incorrect rounding in (a) e.g. 10.97 for *a*
Correct answer only scores both marks.
Note that part (b) can be done without having attempted part (a)

(c)

M1: This mark is awarded for one of:

- substituting $t = 12$ into their $D = "10.96" \times "2.399"^{12}$ and proceeding to a **value** for D
- substituting $t = 12$ into $\log_{10} D = 1.04 + 0.38t$ proceeding to a value for D
- substituting $D = 350000$ into their $D = "10.96" \times "2.399"^t$ and proceeding to a **value** for t
- substituting $D = 350000$ into $\log_{10} D = 1.04 + 0.38t$ and proceeding to a **value** for t

A1: This mark is awarded for one of:

- obtaining D = awrt (£)400 000 and making a (minimal) conclusion e.g. "yes"
- obtaining t = awrt 11.9 or truncated 11.8 and making a (minimal) conclusion e.g. "yes"

For reference the values of *D*/*t* which may be seen are:
D = 398 266.278 (from rounded values of a and b)
D = 398 107.1706 (from full values of a and b)
t = 11.85236… (from rounded values of a and b)
t = 11.85281… (from full values of a and b)

Allow recovery from incorrect rounding in (a) e.g. 10.97 for *a*
Note that part (c) can be done without having attempted part (a)

Acceptable alternative for (c):

$\log_{10} 350000 = 5.54...,\ 1.04 + 0.38 \times 12 = 5.6$ so "yes"

or e.g.

$\log_{10} 350000 = 1.04 + 0.38 \times 12,\ 5.544... < 5.6$ so "yes"

(Condone e.g. $5.54 \approx 5.6$ so "yes")

Score M1 for substituting D = 350 000 and t = 12 into the given equation and evaluating both sides and then A1 for a correct conclusion with lhs evaluated as awrt 5.54 or truncated 5.5 and rhs evaluated as 5.6

Allow obvious slips in the copying of 45 000 and/or 350 000 used as e.g. 4500 or 35000 for the M marks in (b) and (c)

Question Number	Scheme	Marks
4(a)	$\text{f}\left(x\right)=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}$	B1
	$=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2x^2-32+8(x+4)}{(3x-5)(x+4)}$ or e.g. $=\dfrac{2\left(x-4\right)\left(x+4\right)}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2\left(x-4\right)+8}{(3x-5)}$	M1
	$=\dfrac{2x\,\cancel{(x+4)}}{(3x-5)\,\cancel{(x+4)}}=\dfrac{2x}{3x-5}$ *	A1*
		(3)
(b)	$\text{f}'(x)=\dfrac{2\left(3x-5\right)-3\times 2x}{(3x-5)^2}$	M1A1
	$\text{f}'(x)=\dfrac{-10}{(3x-5)^2}$ As $(3x-5)^2>0$ (for $x>2$) then $\text{f}'(x)<0$ Hence f is a decreasing function *	A1cso*
		(3)
(c)	$\text{g}^{-1}(x)=\text{e}^{\frac{x-3}{2}}$	M1A1
	$x\geqslant 3$	B1
		(3)
(d)	$\text{g}\left(\dfrac{2a}{3a-5}\right)=5\Rightarrow 3+2\ln\left(\dfrac{2a}{3a-5}\right)=5$	B1
	$\ln\left(\dfrac{2a}{3a-5}\right)=1\Rightarrow\dfrac{2a}{3a-5}=\text{e}$	M1
	$\dfrac{2a}{3a-5}=\text{e}\Rightarrow a=...$	**d**M1
	$a=\dfrac{5\text{e}}{3\text{e}-2}$	A1
		(4)
(d) Way 2	$\text{gf}\left(a\right)=5\Rightarrow\dfrac{2a}{3a-5}=\text{g}^{-1}\left(5\right)$	B1
	$\dfrac{2a}{3a-5}=\text{g}^{-1}\left(5\right)\Rightarrow\dfrac{2a}{3a-5}=\text{e}$	M1
	$\dfrac{2a}{3a-5}=\text{e}\Rightarrow a=...$	**d**M1
	$a=\dfrac{5\text{e}}{3\text{e}-2}$	A1
		(13 marks)

(a)

B1: $3x^2+7x-20=(3x-5)(x+4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is "recovered".

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2-32}{3x^2+7x-20}+\frac{8}{3x-5}=\frac{(2x^2-32)(3x-5)+8(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)} \quad \textbf{M1}$$

$$=\frac{6x^3+14x^2-40x}{(3x^2+7x-20)(3x-5)}=\frac{2x(3x^2+7x-20)}{(3x-5)(x+4)(3x-5)} \quad \textbf{B1}$$

$$=\frac{2x(3x-5)(x+4)}{(3x-5)(x+4)(3x-5)}=\frac{2x}{3x-5}* \quad \textbf{A1*}$$

Or

$$\frac{2x^2-32}{3x^2+7x-20}+\frac{8}{3x-5}=\frac{(2x^2-32)(3x-5)+8(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)} \quad \textbf{M1}$$

$$=\frac{6x^3+14x^2-40x}{(3x^2+7x-20)(3x-5)}=\frac{2x(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)} \quad \textbf{B1}$$

In this case the B1 is scored for obtaining a factor of $3x^2+7x-20$ in the numerator and denominator

$$=\frac{2x}{3x-5}* \quad \textbf{A1*}$$

(b)

M1: Attempts to differentiate using the product or quotient rule. Award for an expression of the form $\frac{\alpha(3x-5)-\beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1}-\beta x(3x-5)^{-2},\ \alpha,\beta>0$

Condone attempts where $(3x-5)^2$ is expanded.

Alternatively, attempts to write $\frac{2x}{3x-5}$ as $A+\frac{B}{3x-5}$ before differentiating using the chain rule to obtain $\frac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\frac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2>0$ oe e.g. denominator is positive (condone $(3x-5)^2 \geqslant 0$)
- so $f'(x)<0$
- function is decreasing

Some candidates may attempt to differentiate the original f(x) e.g.:

$$f'(x)=\frac{4x\left(3x^2+7x-20\right)-\left(2x^2-32\right)\left(6x+7\right)}{(3x^2+7x-20)^2}-\frac{24}{\left(3x-5\right)^2}$$

Score M1 for:

$$f'(x)=\frac{Ax\left(3x^2+7x-20\right)-\left(2x^2-32\right)\left(Cx+D\right)}{(3x^2+7x-20)^2}-\frac{E}{\left(3x-5\right)^2}$$

and (<u>first</u>) A1 if they reach $\frac{-10}{(3x-5)^2}$

(c)

M1: Rearranges $y=3+2\ln x$ to $x=e^{f(y)}$ or $x=3+2\ln y$ to $y=e^{f(x)}$

A1: Obtains $g^{-1}(x)=e^{\frac{x-3}{2}}$ or equivalent e.g. $g^{-1}(x)=\sqrt{e^{x-3}}$, $g^{-1}(x)=e^{\frac{3-x}{-2}}$ and isw.

Accept $g^{-1}(x)=...$, $g^{-1}=...$, $y=...$ but not e.g. $f^{-1}(x)=...$

B1: Correct domain: $x\geqslant 3$ oe using correct notation e.g. $[3,\infty)$

(d) Condone the miscopy of f(x) for the M marks as long as it has the correct form e.g. $\frac{...a}{...a\pm...}$

and allow use of x instead of a for <u>all</u> marks in (d)

B1: Sets up a valid equation in a e.g. $3+2\ln\left(\frac{2a}{3a-5}\right)=5$

M1: Rearranges to obtain $\ln\left(\frac{2a}{3a-5}\right)=k$ and uses the inverse law of logarithms to remove the ln correctly to reach $\frac{2a}{3a-5}=e^k$ oe

dM1: Rearranges an equation of the form $\frac{2a}{3a-5}=e^k$ to obtain $a=\frac{...e^k}{...e^k\pm...}$ or equivalent.

Depends on the previous method mark.

A1: $(a=)\ \frac{5e}{3e-2}$ oe e.g. $(a=)\ \frac{-5e}{2-3e}$ and apply isw once the correct exact answer is seen.

Accept e^1 for e.

(d) Way 2

B1: Sets up a valid equation in a e.g. $\frac{2a}{3a-5} = \mathrm{g}^{-1}(5)$ seen or implied.

M1: Attempts $\mathrm{g}^{-1}(5)$ and reaches $\frac{2a}{3a-5} = \mathrm{e}^k$ oe

dM1: Rearranges an equation of the form $\frac{2a}{3a-5} = \mathrm{e}^k$ to obtain $a = \frac{...\mathrm{e}^k}{...\mathrm{e}^k \pm ...}$ or equivalent.

Depends on the previous method mark.

A1: $(a =)\ \frac{5\mathrm{e}}{3\mathrm{e}-2}$ oe e.g. $(a =)\ \frac{-5\mathrm{e}}{2-3\mathrm{e}}$ and apply isw once the correct exact answer is seen.

Accept e^1 for e.

<u>Alternative for (d) which has been seen:</u>

$$\mathrm{g}\left(\frac{2a}{3a-5}\right) = 5 \Rightarrow 3 + 2\ln\left(\frac{2a}{3a-5}\right) = 5:$$ **B1 as above**

$$\Rightarrow \ln\left(\frac{2a}{3a-5}\right)^2 = 2 \Rightarrow \left(\frac{2a}{3a-5}\right)^2 = \mathrm{e}^2:$$ **M1 rearranges to reach** $\left(\frac{2a}{3a-5}\right)^2 = \mathrm{e}^{...}$

$$\Rightarrow (9\mathrm{e}^2 - 4)a^2 - 30\mathrm{e}^2 a + 25\mathrm{e}^2 = 0$$

$$\Rightarrow a = \frac{30\mathrm{e}^2 \pm \sqrt{900\mathrm{e}^4 - 100\mathrm{e}^2(9\mathrm{e}^2-4)}}{2(9\mathrm{e}^2-4)}:$$ **dM1 forms and solves 3TQ in *a* (usual rules)**

$$= \frac{5\mathrm{e}}{3\mathrm{e}-2} \text{ oe}:$$ **A1 as above**

Question Number	Scheme	Marks
5(a)	$(A=)8$	B1 **(1)**
(b)	$16=10+"8"\mathrm{e}^{-Bt} \Rightarrow \mathrm{e}^{-Bt}=...$	M1
	$\mathrm{e}^{-45B}="\frac{3}{4}" \Rightarrow -45B=\ln"\frac{3}{4}" \Rightarrow B=...$	**d**M1
	$B=\text{awrt } 0.00639$	A1
		(3)
(c)	$\frac{\mathrm{d}T}{\mathrm{d}t}=-"8"\times\frac{1}{45}\ln\left(\frac{4}{3}\right)\times \mathrm{e}^{-\frac{1}{45}\ln\left(\frac{4}{3}\right)\times 2}=...$	M1
	$=-0.0505$	A1
		(2)
(d)	The temperature has a (lower) limit of $10\,°\mathrm{C}$ **or** $5=10+"8"\mathrm{e}^{-Bt} \Rightarrow \mathrm{e}^{-Bt}=-\frac{5}{"8"}$ e.g. which is not possible or cannot be solved or you cannot find the log of a negative number	B1 **(1)**
		(7 marks)

(a)

B1: $(A=)8$ or award for $(T=)10+8\mathrm{e}^{-Bt}$. Note that "$A=$" is not required, just look for 8.

(b)

M1: Sets $16=10+"8"\mathrm{e}^{-Bt}$ and rearranges to the form $\mathrm{e}^{-Bt}=...$ with or without $t = 45$

dM1: Uses $t = 45$, takes lns of both sides and proceeds to find a numerical expression for B

Depends on the previous method mark.

A1: Achieves awrt 0.00639 or 6.39×10^{-3} or e.g. $\frac{1}{45}\ln\left(\frac{4}{3}\right)$ or $-\frac{1}{45}\ln\left(\frac{3}{4}\right)$ or $\frac{-\ln(0.75)}{45}$ etc.

and apply isw once the correct value is seen. **Correct answer only scores <u>no</u> marks.**

(c)

M1: Differentiates using the chain rule to obtain $...\mathrm{e}^{-Bt}$ (not $...\mathrm{e}^{Bt}$) where … is a constant

and substitutes in $t=2$ to obtain a value for $\frac{\mathrm{d}T}{\mathrm{d}t}$.

Condone $\frac{\mathrm{d}T}{\mathrm{d}t}="8"\mathrm{e}^{-Bt}$ as long as it is clear they think they have found $\frac{\mathrm{d}T}{\mathrm{d}t}$.

If they lose the minus sign in $...\mathrm{e}^{-Bt}$ they obtain ±0.0518… and this scores M0

Note that if there is evidence that $\frac{\mathrm{d}T}{\mathrm{d}t}=...t\mathrm{e}^{-Bt}$ is being used, score M0

A1: -0.0505 cao. Accept 0.0505 **decreasing** or equivalent **<u>following correct work</u>**.

Ignore any units, correct or incorrect)

Correct answer only scores <u>no</u> marks.

(d)

B1: Either states that the (lower) limit of the model is $10\,°\text{C}$ or there is an asymptote at

T = 10 or as t tends to ∞ T tends to 10. But do not allow incorrect statements e.g. the

maximum temperature is 10. Also allow "the maximum the temperature can drop to is 10"

or

shows that the calculation cannot be carried out and makes a statement to that effect.

The working must be correct for their A and/or B where A > 0 and B > 0 and they must reach at least $e^{-Bt} = ...$

Allow comments such as: "which is not possible", "cannot be done", "you cannot find the log of a negative number" etc. but do not allow ambiguous/incorrect statements e.g. "logs cannot be negative", "you cannot have a negative time"

Also accept equivalent arguments e.g. " Ae^{-Bt} would need to be < 0 and this is not possible" (provided their A > 0)

If you are unsure if the reasoning is acceptable or not, use review.

Question Number	Scheme	Marks
6(a)	$\left(\frac{3\pi}{2}, 0\right)$	B1
		(1)
(b)	$\mathrm{f}'(x) = \left(6\cos^2 x\right)\mathrm{e}^{3\sin x} - \left(2\sin x\right)\mathrm{e}^{3\sin x}$	M1A1
	$\mathrm{f}'(x) = \left(6\cos^2 x\right)\mathrm{e}^{3\sin x} - \left(2\sin x\right)\mathrm{e}^{3\sin x}$ $\Rightarrow \left(6\left(1-\sin^2 x\right) - 2\sin x\right)(=0)$	**d**M1
	$\Rightarrow 3\sin^2 x + \sin x - 3 = 0$ oe	A1
		(4)
(c)	$\sin x = \frac{-1+\sqrt{37}}{6} \Rightarrow x = ...$	M1
	$x = 2.131$	A1
		(2)
		(7 marks)

(a)

B1: $\left(\frac{3\pi}{2}, 0\right)$ or accept the identification that $x = \frac{3\pi}{2}$ at R e.g. cos x = 0 so $x = \frac{3\pi}{2}$

Accept $\frac{3\pi}{2}$ written on the diagram at the point R or just $\frac{3\pi}{2}$ seen as the answer as long as there is no evidence that y is anything other than zero. Must be exact.

(b)

M1: Attempts the product rule. Award for the form $\mathrm{f}'(x) = \pm\alpha\cos^2 x\mathrm{e}^{3\sin x} \pm \beta\sin x\mathrm{e}^{3\sin x}$

A1: Correct derivative in any form

dM1: Attempts to use the identity $\pm\sin^2 x \pm \cos^2 x = \pm 1$ to produce a 3 term quadratic expression in sin x which may be seen embedded but with the $\mathrm{e}^{3\sin x}$ factored.

e.g. $\mathrm{e}^{3\sin x}\left(6\left(1-\sin^2 x\right) - 2\sin x\right)$

Depends on the previous method mark.

A1: $3\sin^2 x + \sin x - 3 = 0$ or any integer multiple of this equation e.g. $6 - 6\sin^2 x - 2\sin x = 0$ and apply isw once a correct equation is seen.

Must be "extracted" but allow if seen, used or implied in part (c) and allow recovery from missing brackets if the quadratic expression is extracted correctly.

Beware:

$\mathrm{f}'(x) = \left(-6\cos^2 x\right)\mathrm{e}^{3\sin x} + \left(2\sin x\right)\mathrm{e}^{3\sin x} = 0$ leading to $3\sin^2 x + \sin x - 3 = 0$ scores

M1A0dM1A0

But allow full recovery in (c)

(c)

M1: Solves their 3-term quadratic in sin x (allow any method including calculator) and proceeds to find a value of x using $\sin^{-1}$(their sin x) where $|\sin x| < 1.$ You may need to check their value.

The attempt to solve their quadratic may be implied by at least 1 correct value for sin x (allow 2dp if inexact).

A1: awrt 2.131 only and no other values offered.

Correct answer only scores no marks.

Question Number	Scheme	Marks
7(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{16}{3}(3x-k)^{-2}$	M1A1
		(2)
(b)	$-\frac{16}{3}(3x-k)^{-2}=-12\Rightarrow(3x-k)^{2}=...$	M1
	$3-k=\text{"}\pm\frac{2}{3}\text{"}\Rightarrow k=...$	**d**M1
	$k=\frac{7}{3},\ \frac{11}{3}$	A1
		(3)
(c)	$y=\frac{16}{9\left(3(1)-\frac{7}{3}\right)}$	M1 (B1 on ePEN)
	$y-\text{"}\frac{8}{3}\text{"}=\frac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
		(3)
(d)	$\int\frac{16}{9(3x-k)}\mathrm{d}x=\frac{16}{27}\left[\ln(3x-k)\right]$	M1
	$=\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]$	A1ft
	$\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]_1^3=\frac{16}{27}\left(\ln\left(3(3)-\frac{7}{3}\right)-\ln\left(3(1)-\frac{7}{3}\right)\right)$	**d**M1
	$=\frac{16}{27}\ln(10)$	A1
		(4)
		(12 marks)

(a)

M1: Attempts to differentiate to the form $A(3x-k)^{-2}$ oe e.g. $A(27x-9k)^{-2}$

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{16}{3}(3x-k)^{-2}$ oe

e.g. $-\frac{16}{3(3x-k)^2}$ or $-\frac{16}{3(9x^2-6kx+k^2)}$ or $-\frac{16}{27x^2-18kx+3k^2}$

but **not**

$-432(27x-9k)^{-2}$ or $-\frac{432}{(27x-9k)^2}$

as there is a common factor in the numerator and denominator.

(b)

M1: Sets their derivative of the form $A(3x-k)^{-2}$ (or equivalent) equal to -12 (not 12) and rearranges to $...(3x-k)^2 = ...$ or equivalent e.g. $...(27x-9k)^2 = ...$ or $...(9x^2-6kx+k^2) = ...$

Condone poor squaring e.g. allow $...(9x^2+k^2) = ...$

May have already substituted $x = 1$

dM1: **<u>Depends on having obtained $A < 0$</u> (otherwise the equation has no real solutions):**
Substitutes $x = 1$ and solves to find 2 values for k.
If the $(3x-k)^2$ is expanded then the usual rules apply for solving a 3TQ and allow using a calculator. FYI correct 3TQ is $9k^2-54k+77=0$
Depends on the previous method mark.

A1: Achieves $k=\frac{7}{3}$ and $k=\frac{11}{3}$ from a correct method. Accept equivalent exact fractions or recurring decimals $2.\dot{3},\ 3.\dot{6}$ but not rounded decimals e.g. 2.33, 3.67

(c) **Note we are scoring the first mark as an M mark not a B mark.**

M1(B1 on ePEN):
Uses a value of k from part (b) (or a 'made up' k) and $x = 1$ to find the value of y at P.

dM1: Attempts to find the equation of the normal using their y-coordinate with the gradient $\frac{1}{12}$.
If they use $y=mx+c$ they must proceed as far as $c = ...$
Depends on the previous method mark.

A1: $12y-x-31=0$ or any equivalent integer multiple of this equation.
Note if $k=\frac{11}{3}$ is used they should get $x - 12y - 33 = 0$ and would generally score 110

(d)

M1: Integrates to the form $B\ln(3x-k)$ oe e.g. $B\ln\alpha(3x-k)$

A1ft: $\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]$ which may be unsimplified and isw once correct integration is seen.
Follow through their k and allow the letter k and allow if their k is not less than 3.
Ignore any reference to $+c$.
You may need to check their integration carefully.

E.g. $\frac{16}{27}\left[\ln(27x-21)\right]$ is also correct (for $k=\frac{7}{3}$)

Ignore any spurious integral signs after a correct integral is seen.

dM1: Substitutes in the limits 3 and 1 and subtracts either way round. Must have a numeric *k* now.

It is dependent on the first method mark.

A1: $\frac{16}{27}\ln(10)$ or exact equivalent e.g. $\frac{32}{54}\ln(10)$

Use of an incorrect *k* in (d) scores a maximum of M1A1ft dM1A0

Note that in part (d), some candidates may use substitution e.g.

$$u = 3x - k \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3 \Rightarrow \int \frac{16}{9(3x-k)}\,\mathrm{d}x = \frac{16}{9}\int \frac{1}{u}\,\frac{1}{3}\,\mathrm{d}u = \frac{16}{27}\ln u$$

Score M1 for integrating to the correct form e.g. $k\ln u$

and A1 for $\frac{16}{27}\ln u$ **following through their *k* or the letter *k* as above**

then dM1 for

$$\left[\frac{16}{27}\ln u\right]_{\frac{2}{3}}^{\frac{20}{3}} = \frac{16}{27}\ln\frac{20}{3} - \frac{16}{27}\ln\frac{2}{3} \text{ or } \left[\frac{16}{27}\ln\left(3x - \frac{7}{3}\right)\right]_1^3 = \frac{16}{27}\ln\frac{20}{3} - \frac{16}{27}\ln\frac{2}{3}$$

i.e. applies the correct changed limits or reverts to *x* and uses 3 and 1

$$= \frac{16}{27}\ln(10) \quad \textbf{A1}$$

Note if $k = \frac{11}{3}$ is used they should get $= \frac{16}{27}\ln(8)$ and would generally score 1110

<u>Note:</u>

If you see any responses where the denominator in part (a) is expanded incorrectly

e.g. $\frac{16}{9(3x-k)} = \frac{16}{27x-k}$

and candidates persist with this incorrect expansion then send to review.

A typical response with an expanded denominator:

Question Number	Scheme	Marks
7(a)	$y=\dfrac{16}{9(3x-k)}=\dfrac{16}{27x-9k}\Rightarrow\dfrac{dy}{dx}=-\dfrac{432}{(27x-9k)^2}$	M1A0
(b)	$-\dfrac{432}{(27x-9k)^2}=-12\Rightarrow 12(27x-9k)^2=432$	M1
	$12(27x-9k)^2=432\Rightarrow(27x-9k)^2=36\Rightarrow 27-9k=\pm6\Rightarrow k=...$	**d**M1
	$k=\dfrac{7}{3},\dfrac{11}{3}$	A1
(c)	$y=\dfrac{16}{27-21}$	M1 (B1 on ePEN)
	$y-"\dfrac{8}{3}"=\dfrac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
(d)	$\displaystyle\int\frac{16}{27x-9k}dx=\frac{16}{27}[\ln(27x-9k)]$	M1
	$=\dfrac{16}{27}\left[\ln(27x-21)\right]$	A1ft
	$\dfrac{16}{27}\left[\ln(27x-21)\right]_1^3=\dfrac{16}{27}\left(\ln(27(3)-21)-\ln(27-21)\right)$	**d**M1
	$=\dfrac{16}{27}\ln(10)$	A1

Question Number	Scheme	Marks
8(a)(i)	$\left(\frac{b}{2},\ a\right)$	B1B1
(ii)	$(0,\ a-b)$	B1
(iii)	$\left(\frac{b-a}{2},\ 0\right)$ and $\left(\frac{a+b}{2},\ 0\right)$	B1B1
		(5)

(a) (i)

B1: One correct coordinate $x=\frac{1}{2}b$ or $y = a$

B1: $\left(\frac{b}{2},\ a\right)$ or $x=\frac{1}{2}b$ and $y = a$

May be seen on the sketch.

(ii)

B1: $(0,\ a-b)$ **or** $x=0$ and $y = a - b$ **or** just $y = a - b$ without the $x = 0$

May be seen on the sketch and if so, allow just $a - b$ marked in the correct place.

If it is on the sketch, condone $(a-b,\ 0)$ marked in the **correct place**.

If more than 1 point is offered and no clear decision is made then score B0

(iii)

B1: $\left(\frac{b-a}{2},\ 0\right)$ **or** $\left(\frac{a+b}{2},\ 0\right)$ Allow just $x=\frac{b-a}{2}$ or $x=\frac{a+b}{2}$ without the $y = 0$

May be seen on the sketch and allow just $\frac{b-a}{2}$ or $\frac{a+b}{2}$ marked in the **correct place**.

If they are on the sketch, condone $\left(0,\ \frac{b-a}{2}\right)$ or $\left(0,\ \frac{a+b}{2}\right)$ marked in the **correct place**.

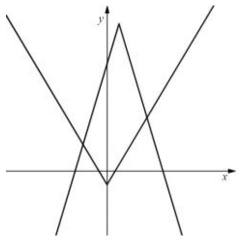
Allow equivalent expressions e.g. $\frac{-a-b}{-2}$ for $\frac{a+b}{2}$ or e.g. $\frac{a-b}{-2}$ for $\frac{b-a}{2}$

B1: $\left(\frac{b-a}{2},\ 0\right)$ **and** $\left(\frac{a+b}{2},\ 0\right)$ Allow just $x=\frac{b-a}{2}$ or $x=\frac{a+b}{2}$ without the $y = 0$

May be seen on the sketch and allow just $\frac{b-a}{2}$ and $\frac{a+b}{2}$ marked in the **correct place**.

If they are on the sketch, condone $\left(0,\ \frac{b-a}{2}\right)$ and $\left(0,\ \frac{a+b}{2}\right)$ marked in the **correct place**.

Allow equivalent expressions e.g. $\frac{-a-b}{-2}$ for $\frac{a+b}{2}$ or e.g. $\frac{a-b}{-2}$ for $\frac{b-a}{2}$

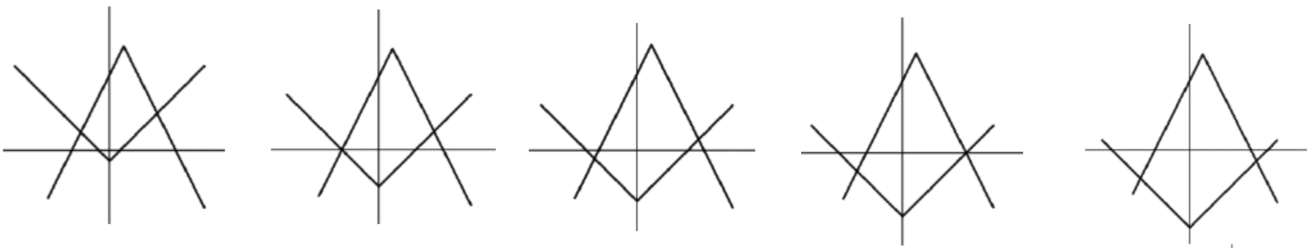
(b)		B1B1
		(2)
(c)	$-x-1=2x+a-b,\ x=-3 \Rightarrow 2=-6+a-b$ or $x-1=a+b-2x,\ x=5 \Rightarrow 5-1=a+b-10$	M1
	$-x-1=2x+a-b,\ x=-3 \Rightarrow 2=-6+a-b$ and $x-1=a+b-2x,\ x=5 \Rightarrow 5-1=a+b-10$	dM1 (A1 on ePEN)
	$\begin{matrix} a-b=8 \\ a+b=14 \end{matrix} \Rightarrow a=\ldots \text{ or } b=\ldots$	**dd**M1
	$a=11,\ b=3$	A1
		(4)
		(11 marks)

(b)

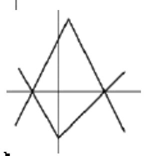
B1: A V shaped graph anywhere on the set of axes

B1: Correct shape with a minimum point on the *y*-axis below the *x*-axis and with an intention for the graph to be symmetrical about the *y*-axis. The left branch of $y=|x|-1$ must intersect the left branch of $y=a-|2x-b|$ and the right branch of $y=|x|-1$ must intersect the right branch of $y=a-|2x-b|$

These are all acceptable for both marks:



But the intersections of the branches must not both be on the *x*-axis e.g.



Some candidates may use Figure 2 to do their sketch which is acceptable but if both Diagram 1 and Figure 2 are used and neither is “rejected” then Diagram 1 takes precedence.

If they draw a sketch of $y=|x|-1$ on their own axes, only the first B mark is available unless they draw the given graph as well in which case the 2nd B is available.

(c) Note that $x=-3 \Rightarrow y=2$ and $x=5 \Rightarrow y=4$ **and may be used to find the equations.**

M1: Forms one correct equation in *a* and *b* only with modulus signs removed, simplified or unsimplified. There may be several equations so you will need to check if any of them are correct.

dM1(A1 on ePEN):

Forms 2 correct equations in *a* and *b* only with modulus signs removed, simplified or unsimplified. There may be several equations so you will need to check if 2 of them are correct.

Note that for their second equation, some candidates may substitute in for *a* or *b* from their 1st equation but may make a slip when rearranging. Condone this if the 2nd equation is otherwise correct.

ddM1: Solves the correct equations in *a* and *b* e.g. $8=a-b$ and $14=a+b$ or equivalent to find a value for *a* or a value for *b*.

A1: $a=11,\ b=3$ (Correct answers with no working scores no marks)

Question Number	Scheme	Marks
9(a)	$\dfrac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta}=(2+\sec 2\theta)(\cos\theta-\sin\theta)$ $\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)(\cos\theta-\sin\theta)\left(\cos\theta+\sin\theta\right)$ **or** $\Rightarrow 3\sin\theta\cos\theta=(2+\sec 2\theta)\cos 2\theta$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$	**d**M1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$ $\Rightarrow \dfrac{3}{2}\sin 2\theta=2\cos 2\theta+1\Rightarrow 3\sin 2\theta-4\cos 2\theta=2\ *$	A1*
		(3)

(a)

M1: Attempts to use $\sin 2\theta=2\sin\theta\cos\theta$ **or** $\cos 2\theta=\pm\cos^2\theta\pm\sin^2\theta$
Accept going from $(\cos\theta-\sin\theta)\left(\cos\theta+\sin\theta\right)$ to $\cos 2\theta$

dM1: Attempts to use $\sin 2\theta=2\sin\theta\cos\theta$ **and** $\cos 2\theta=\pm\cos^2\theta\pm\sin^2\theta$
Accept going from $(\cos\theta-\sin\theta)\left(\cos\theta+\sin\theta\right)$ to $\cos 2\theta$
Depends on the previous method mark.

A1*: Uses the identity $\sec 2\theta=\dfrac{1}{\cos 2\theta}$ to achieve an equation in just $\sin 2\theta$ and $\cos 2\theta$ and proceeds to the given answer with all previous stages of working seen.
Note we would accept going from e.g. $6\sin\theta\cos\theta=4\cos 2\theta+2$ to the given answer.

You can condone e.g. a missing θ or a missing bracket but withhold the A mark if there are clear errors such as a missing "2" or a missing pair of brackets that are not recovered.

There are various alternatives in part (b) and in each case:

- **the first 3 marks are for obtaining a suitable equation**
- **the final 2 marks are for solving their equation**

Condone the use of a variable other than *x* in (b) e.g. θ

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **or** $(\alpha=)\ \tan^{-1}\left(\frac{4}{3}\text{ or }\frac{3}{4}\right)=...$	M1
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **and** $(\alpha=)\ \tan^{-1}\left(\frac{4}{3}\text{ or }\frac{3}{4}\right)=...$	**d**M1
	$3\sin 2x-4\cos 2x=2\Rightarrow 5\sin(2x-0.927)=2$ **or** $3\sin 2x-4\cos 2x=2\Rightarrow 5\cos(2x+0.644)=-2$	A1
	$5\sin(2x-0.927)=2\Rightarrow x=\dfrac{\sin^{-1}\frac{2}{5}+0.927}{2}$ **or** $5\cos(2x+0.644)=-2\Rightarrow x=\dfrac{\cos^{-1}\frac{-2}{5}-0.644}{2}$	**dd**M1
	$x=$ awrt 3.81	A1
		(5)
		(8 marks)

Note that is it not necessary to state the form $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$ for the first 2 marks

M1: $R=\sqrt{3^2+4^2}\ (=5)$ Condone obtaining ± 5

or $(\alpha=)\ \tan^{-1}\left(\pm\frac{4}{3}\right),\ \tan^{-1}\left(\pm\frac{3}{4}\right),\ \cos^{-1}\left(\pm\frac{3}{"5"}\right),\ \sin^{-1}\left(\pm\frac{4}{"5"}\right),\ \cos^{-1}\left(\pm\frac{4}{5}\right),\ \sin^{-1}\left(\pm\frac{3}{5}\right)$

dM1: $R=\sqrt{3^2+4^2}\ (=5)$ Condone obtaining ± 5

and $(\alpha=)\ \tan^{-1}\left(\pm\frac{4}{3}\right),\ \tan^{-1}\left(\pm\frac{3}{4}\right),\ \cos^{-1}\left(\pm\frac{3}{"5"}\right),\ \sin^{-1}\left(\pm\frac{4}{"5"}\right),\ \cos^{-1}\left(\pm\frac{4}{5}\right),\ \sin^{-1}\left(\pm\frac{3}{5}\right)$

Depends on the previous method mark.

A1: Correct equation: $5\sin(2x-0.927)=2$ **or** $5\cos(2x+0.644)=-2$ which may be implied.
Accept awrt 0.93 or 0.64 for "α".
Do not allow e.g. $\pm 5\sin(2x-0.927)=2$ or $\pm 5\cos(2x+0.644)=-2$ unless they recover by solving the correct equation.

ddM1: Attempts to solve their equation of the correct form e.g. with ($2x$ ± …) as the argument and proceeds to find a value for *x*.
The order of operations should be correct <u>for their equation</u> but condone $\pm"0.927"$ or $\pm"0.644"$

e.g. $x=\dfrac{\left(\sin^{-1}\left(\frac{2}{"5"}\right)\right)\pm"0.927"}{2}$ or $x=\dfrac{\cos^{-1}\frac{-2}{"5"}\pm 0.644}{2}$

You may need to check their angle on your calculator.
It is dependent on both of the previous method marks.

(b)	Way 2 Squaring	
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 3\sin 2x = 2 + 4\cos 2x$ $\Rightarrow 9\sin^2 2x = 4 + 16\cos 2x + 16\cos^2 2x$ or $3\sin 2x - 4\cos 2x = 2 \Rightarrow 3\sin 2x - 2 = 4\cos 2x$ $\Rightarrow 9\sin^2 2x - 12\sin 2x + 4 = 16\cos^2 2x$	M1
	$9\left(1-\cos^2 2x\right) = 4 + 16\cos 2x + 16\cos^2 2x$ $\Rightarrow 25\cos^2 2x + 16\cos 2x - 5 = 0$ or $9\sin^2 2x - 12\sin 2x + 4 = 16\left(1-\sin^2 2x\right)$ $\Rightarrow 25\sin^2 2x - 12\sin 2x - 12 = 0$	**d**M1A1
	$25\cos^2 2x + 16\cos 2x - 5 = 0 \Rightarrow \cos 2x = \dfrac{-8 \pm 3\sqrt{21}}{25}\left(0.23,\ -0.87\right)$ or $25\sin^2 2x - 12\sin 2x - 12 = 0 \Rightarrow \sin 2x = \dfrac{6 \pm 4\sqrt{21}}{25}\left(0.97,\ -0.49\right)$ $\Rightarrow x = ...$	**dd**M1
	$x =$ awrt 3.81	A1

A1: $x =$ awrt 3.81 and no other values in range. ($x = \pi + 0.669$ scores A0)

M1: Isolates 1 trig term on one side and then attempts to square both sides and multiply out the brackets. Do **not** condone poor squaring for this mark. e.g. do not condone

$$3\sin 2x = 2 + 4\cos 2x \Rightarrow 9\sin^2 2x = 4 + 16\cos^2 2x$$

dM1: Attempts to use $\pm\sin^2 2x \pm \cos^2 2x = \pm 1$ and proceeds to a <u>3-term</u> quadratic equation in either $\sin 2x$ or $\cos 2x$
Depends on the previous method mark.

A1: $25\cos^2 2x + 16\cos 2x - 5 = 0$ or $25\sin^2 2x - 12\sin 2x - 12 = 0$ oe

ddM1: Attempts to solve their 3TQ in either $\sin 2x$ or $\cos 2x$ and proceeds to find value for x by taking $\sin^{-1}$ or $\cos^{-1}$ and dividing by 2.
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator. It is dependent on both of the previous method marks.
You may need to check their angle on your calculator.
It is dependent on both of the previous method marks.

A1: awrt 3.81 and no other values in range. ($x = \pi + 0.669$ scores A0)

(b)	Way 3 Using double angle formulae	
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 6\sin x\cos x - 4\cos^2 x + 4\sin^2 x = 2$	M1
	$6\sin x\cos x - 4\cos^2 x + 4\sin^2 x = 2 \Rightarrow 6\tan x - 4 + 4\tan^2 x = 2\sec^2 x$ $\Rightarrow 6\tan x - 4 + 4\tan^2 x = 2\left(1+\tan^2 x\right)$ $\Rightarrow \tan^2 x + 3\tan x - 3 = 0$	**d**M1A1
	$\Rightarrow \tan^2 x + 3\tan x - 3 = 0 \Rightarrow \tan x = \dfrac{-3 \pm \sqrt{21}}{2}\left(0.79,\ -3.79\right)$ $\Rightarrow x = ...$	**dd**M1
	$x =$ awrt 3.81	A1

M1: Attempts to use $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm\sin^2 x \pm \cos^2 x$ oe
e.g. $\cos 2x = \pm 1 \pm 2\cos^2 x$

dM1: Divides by $\cos^2 x$ and attempts to use $\sec^2 x = \pm 1 \pm \tan^2 x$ and proceeds to a <u>3-term</u> quadratic equation in tan *x*. Alternatively may divide by $\sin^2 x$ and proceed to a <u>3-term</u> quadratic equation in cot *x*.
It is dependent on the previous method mark.

A1: $\tan^2 x + 3\tan x - 3 = 0$ oe $3\cot^2 x - 3\cot x - 1 = 0$

ddM1: Attempts to solve their 3TQ in tan *x* (or cot *x*) and proceeds to find a value for *x*.
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator. It is dependent on both of the previous method marks.
You may need to check their angle on your calculator.
It is dependent on both of the previous method marks

A1: awrt 3.81 and no other values in range. ($x = \pi + 0.669$ scores A0)

(b)	Way 4 Divides by cos2x and squares	
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 3\tan 2x - 4 = 2\sec 2x$ $\Rightarrow 9\tan^2 2x - 24\tan 2x + 16 = 4\sec^2 2x$	M1
	$\Rightarrow 9\tan^2 2x - 24\tan 2x + 16 = 4(1+\tan^2 2x)$ $\Rightarrow 5\tan^2 2x - 24\tan 2x + 12 = 0$	**d**M1A1
	$5\tan^2 2x - 24\tan 2x + 12 = 0 \Rightarrow \tan 2x = \frac{12 \pm 2\sqrt{21}}{5}(4.23,\ 0.57)$ $\Rightarrow x = ...$	**dd**M1
	$x =$ awrt 3.81	A1

M1: Divides by cos2x and then attempts to square both sides and multiply out the brackets. Do not condone poor squaring for this mark e.g. $(3\tan 2x - 4)^2 = 9\tan^2 2x + 16$

dM1: Attempts to use $\sec^2 2x = \pm 1 \pm \tan^2 2x$ and proceeds to a <u>3-term</u> quadratic equation in tan 2x
It is dependent on the previous method mark.

A1: $5\tan^2 2x - 24\tan 2x + 12 = 0$ oe

ddM1: Attempts to solve their 3TQ in tan 2x and proceeds to find a value for x by taking $\tan^{-1}$ and dividing by 2.
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator. It is dependent on both of the previous method marks.
You may need to check their angle on your calculator.
It is dependent on both of the previous method marks.

A1: awrt 3.81 and no other values in range. ($x = \pi + 0.669$ scores A0)

Other methods may be seen.
Use review if you are not sure if an attempt deserves credit.